$$\therefore x = \frac{-[(4m^3n-m^4+n^4)^2-16m^2n^2(m^4-n^4)]^2}{16mn(m^4-n^4)(4m^3n-m^4+n^4)(m^4-n^4-4mn^3)(m^4-n^4+4mn^3)}$$

$$=\pm\frac{A^2}{16mn(m^4-n^4)B}, \text{ suppose.}$$

x is $\pm$ according as px, qx, rx is $\mp$.

$\therefore (m^2+n^2)x^2\pm(m^2+n^2)x$

$$=\left[\frac{A}{16mn(m^2-n^2)B}\right]^2[16m^2n^4(n^2-2m^2)+(8mn^3+n^4-m^4)(m^4-n^4)]^2. (6).$$

$(m^2-n^2)x^2\pm(m^2-n^2)x$

$$=\left[\frac{A}{16mn(m^2+n^2)B}\right]^2[16m^2n^4(n^2+2m^2)-(8mn^3+m^4-n^4)(m^4-n^4)]^2. (7).$$

$4m^2n^2x^2\pm2mnx$

$$=\left[\frac{A}{8(m^4-n^4)B}\right][3(m^4-n^4)^2-8m^3n(m^4-n^4)-16m^2n^6]^2. (8).$$

m and n can have any values that make B positive. Let $m=2$, $n=1$; $A=671$, $B=2737$.

(6) gives $(m^2+n^2)^2x^2\pm(m^2+n^2)x=(290543/262752)^2$.

(7) gives $(m^2-n^2)^2x^2\pm(m^2-n^2)x=(74481/437920)^2$.

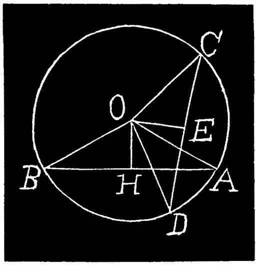
(8) gives $4m^2n^2x^2)\pm2mnx=(234179/328440)^2$.

AVERAGE AND PROBABILITY.

191. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Two random lines cut a given circle. What is the chance that they intersect within the circle?

II. Solulion by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.



Let AB, CD be the random lines, $\angle AOH=\theta$, $\angle COE=\phi$, $\angle EOH=\psi$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of ϕ, 0 and θ; of ψ, $\theta-\phi$ and $\theta+\phi$ for favorable cases, and 0 and π for total cases.

Hence the chance is

$$p=\frac{\int_0^{\frac{1}{2}\pi}\int_0^{\theta}\int_{\theta-\phi}^{\theta+\phi}d\theta\, d\phi\, d\psi}{\int_0^{\frac{1}{2}\pi}\int_0^{\theta}\int_0^{\pi}d\theta\, d\phi\, d\psi}$$

$$=\frac{8}{\pi^3}\int_0^{\frac{1}{2}\pi}\int_0^{\theta}\int_{\theta-\phi}^{\theta+\phi}d\theta\, d\phi\, d\psi=\frac{16}{\pi^3}\int_0^{\frac{1}{2}\pi}\int_0^{\theta}\phi\, d\theta\, d\phi=\frac{8}{\pi^3}\int_0^{\frac{1}{2}\pi}\theta^2 d\theta=\tfrac{1}{3}.$$

One of our readers has again called up the question of the correctness of the two solutions, saying that he is unable to decide which is correct. We shall continue to repeat onr answer as long as this question is asked. There is no such thing as *the correct solution* of the two published solutions—one is just as correct as the other. Both are correct when the law of distribution of the events assumed are granted. To these two solutions might be added an indefinite number of other solutions of equal merit. The fact of the matter is, that the problem is stated in the *indefinite* form, and when thus stated admits of an indefinite number of solutions. There is nothing to prevent one from assuming that the lines are drawn through each of two points taken at random within the circle, making it the equivalent of problem 5450 of the *Educational Times*, as was referred to by our reader. Were one to add to the problem referred to in the *Educational Times* the law of distribution of the points, then this problem would be definite, and there could be only one correct solution possible. ED. F.

MISCELLANEOUS.

169. Proposed by E. D. ROE, Ph. D., Syracuse University, Syracuse, N. Y.

Find the value for all finite values of k of

$$\lim_{x \doteq \infty}\left[x^k \log\left(\frac{e^x+1}{e^x-1}\right)\right].$$

Solution by the PROPOSER.

1. If x is positive, it will be useful to write,

$$x^k\log\left[\frac{e^x+1}{e^x-1}\right]=e^{-x}e^x x^k\log\left[\frac{1+e^{-x}}{1-e^{-x}}\right]=\frac{x^k}{e^x}\log\left[\left(1+\frac{1}{e^x}\right)^{e^x}\left(1-\frac{1}{e^x}\right)^{-e^x}\right].$$

Therefore, $\underset{x\doteq\infty}{L}\, x^k\log\left[\dfrac{e^x+1}{e^x-1}\right]=\log e^2.\underset{x\doteq\infty}{L}\dfrac{x^k}{e^x}=2\,L\dfrac{x^k}{e^x}.$

Since $\dfrac{x^k}{e^x}=\dfrac{x^k}{1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\ldots}$, $\underset{x\doteq\infty}{L}\dfrac{x^k}{e^x}=0$, for all finite values of k, and

therefore $\underset{x\doteq\infty}{L}\, x^k\log\left[\dfrac{e^x+1}{e^x-1}\right]=0.$

2. If x is negative, $\underset{x\doteq\infty}{L}\, x^k\log\left[\dfrac{e^x+1}{e^x-1}\right]=\underset{x\doteq\infty}{L}\, x^k.\log(-1)=\infty.$

Also solved by G. B. M. Zerr, J. Scheffer, and S. A. Corey.